Generalized Global Symmetries in Graded Target Space and Anomalies

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Outline

- The usual description of T-duality in string theory

$$E = \frac{1}{2}\left[\frac{\alpha'}{R^2}n^2 + \frac{R^2}{\alpha'}m^2\right] \qquad \left(R \leftrightarrow \frac{\alpha'}{R}, \qquad m \leftrightarrow n\right)$$

  Exchange of winding energy $m^2 R^2$ and center of mass energy

  However, T-duality can be understood as a feature of worldsheet string theory being a sigma model

  – Can we extend the concept of T-duality to another type of fields, p-forms, mixed tensor field theories, e.g. gravitons, or (2,1) Curtright fields?

  – Is there an off-shell T-duality for gravitons?

- Higher form global symmetries and the t'Hooft anomalies (Gaiotto, Kapustin, Seiberg, Willett (2014)

  – To understand generalized sym. in relation to T-dual pictures

  – Non-linear sigma model for higher-form theories as the action describing Nambu-Goldstone mode of broken phase of higher global symmetries? (Cordova, Dumitrescu, Intriligator (2018))

T-DUALITY

Let's consider a sigma model for a single compact scalar $X(\sigma^\mu)$ with $\sigma^\mu, \mu = 0, 1$ worldsheet coordinates

$$S[X] = -\frac{1}{2}\int_{\Sigma_2} R^2 \, \mathrm{d}X \wedge * \, \mathrm{d}X$$

$X : \Sigma_2 \to S^1_R$

Scalar field is the pullback $X(\sigma) = X^*(x)$ with $x \sim x + 2\pi R$ circle coordinate

Equivalently:

$$\mathcal{S}[F, \widehat{X}] = -\frac{1}{2}\int_{\Sigma_2} R^2 \, F \wedge * \, F + \int_{\Sigma_2} F \wedge \mathrm{d}\widehat{X}$$

EOM for $\widehat{X}$: $\quad dF = 0 \quad$ (locally $F = dX$)

EOM for F : $\quad F = \frac{1}{R^2} * \mathrm{d}\widehat{X} \quad$ Duality Relation

Dual action:

$$\tilde{\mathcal{S}}[\widehat{X}] = -\frac{1}{2}\int_{\Sigma_2} \frac{1}{R^2}\, \mathrm{d}\widehat{X} \wedge * \mathrm{d}\widehat{X}$$

$\widehat{X} : \Sigma_2 \to S^1_{1/R}$

$$S[X] \xleftarrow{\widehat{X} \text{ on-shell}} \mathcal{S}[F, \widehat{X}] \xrightarrow{F \text{ on-shell}} \widetilde{S}[\widehat{X}]$$

Global symmetry and anomaly

Theory has a $U(1)_e \times U(1)_m$ global symmetry
There are two types of ordinary conserved vector currents

$$J_e = R^2 F \quad \text{and} \quad J_m = *F$$

Let's consider coupling to background fields A, $\hat{A}$ ($A \wedge *J_e$ and $\hat{A} \wedge *J_m$)
Under background gauge transformation, we have

$$\delta A = d\epsilon \qquad \delta\hat{A} = d\hat{\epsilon} \qquad \text{and} \qquad \delta X = \epsilon$$

$$S[X, A, \widehat{A}] = -\frac{1}{2}\int_{\Sigma_2} R^2(\mathrm{d}X - A)\wedge *(\mathrm{d}X - A) + \int_{\Sigma_2} \widehat{A}\wedge \mathrm{d}X$$

$$\delta S = \int_{\Sigma_2} \epsilon \wedge d\hat{A}$$

It signals a mixed 't Hooft anomaly between the momentum and winding global symmetries that prevents them from being gauged simultaneously.
Anomalous term is obtained from a 4D anomaly polynomial

$$\mathcal{I}_4 = \int_{\Sigma_4} \mathrm{d}A \wedge \mathrm{d}\widehat{A}$$

Can we gauge $U(1)_m$ instead?

Let's write the parent action

$$\mathcal{S}[F,\widehat{X},A,\widehat{A}] = -\frac{1}{2}\int_{\Sigma_2} R^2(F-A)\wedge *(F-A) + \int_{\Sigma_2} F\wedge(\mathrm{d}\widehat{X}-\widehat{A})$$

EOM for F : $*(F-A) = \frac{1}{R^2}(\mathrm{d}\widehat{X}-\widehat{A})$

T-dual action:

$$S[\widehat{X},A,\widehat{A}] = -\frac{1}{2}\int_{\Sigma_2}\frac{1}{R^2}(\mathrm{d}\widehat{X}-\widehat{A})\wedge *(\mathrm{d}\widehat{X}-\widehat{A}) + \int_{\Sigma_2} A\wedge \mathrm{d}\widehat{X} - \int_{\Sigma_2} A\wedge\widehat{A}$$

$$\delta S = \int_{\Sigma_2}\epsilon\wedge d\hat{A}$$

Anomaly matching in T-dual pictures as expected for t'Hooft anomaly

Question 1: Can it be generalized to non-linear theories?

Outline

Global symmetries

$$S[X] = -\frac{1}{2}\int_{\Sigma_2}\Big(G_{ij}(X,\mathrm{d}X)\,\mathrm{d}X^i \wedge *\,\mathrm{d}X^j + B_{ij}(X,\mathrm{d}X)\,\mathrm{d}X^i \wedge \mathrm{d}X^j\Big)$$

Here $X : \Sigma_2 \to M$

– Components of X: scalar fields X^i with $i = 1, ..., dim(M)$

– G_{ij}, B_{ij} are background fields

– Extension: couplings depends on worldsheet coordinates σ^μ through both X^i and dX^i

Global symmetries: (If G_{ij}, B_{ij} are constant or functions of X only)

$$\delta X^i = \rho^i_a(X)\,\epsilon^a \qquad\qquad (\rho = \rho^i\partial_i = \rho^i_a t^a \partial_i)$$

if

$$\mathcal{L}_{\rho_a} G = 0 \qquad \text{and} \qquad \mathcal{L}_{\rho_a} B = \mathrm{d}\beta_a \quad \Leftrightarrow \quad \mathcal{L}_{\rho_a} H = 0$$

The global symmetries of the theory are Killing isometries of the target space geometry specified by G and H

Comment on the dual picture:

Upon dualization G_{ij}, B_{ij} transform to $\tilde{G}_{ij}, \tilde{B}_{ij}$ s.t. generalized metric $E_{ij} = G_{ij} + B_{ij}$ transform as

$$\widetilde{E}_{mn} = E_{mn} - E_{ma} E^{ab} E_{bn} \,, \quad \widetilde{E}_{ma} = E_{mb} E^{ba} \,, \quad \widetilde{E}_{ab} = E^{ab}$$

$X^i = (X^m, X^a)$ with a number of spatial Killing directions

Fractional linear transformation of E_{ij} under duality group $O(d, d; \mathbb{R})$

This is the duality group for $(p = 2k)$-form in (2p+2) dimensions

Jet space prolongation

Now let's consider $G_{ij}(X^i, dX^i)$ with independent X^i and dX^i

To do so we use: Variational Bicomplex

Extension of configuration space to jet space

Let's consider a smooth fibre bundle (E, π, Σ) with base Σ and fibre M s.t. $\pi : E \to \Sigma$

The first jet manifold of π is the set

$$J^1\pi : \{j^1_p X : p \in \Sigma, X \in \Gamma_p(\pi)\}$$

$j^1_p X$ is an equivalence class of 1-equivalent local sections, i.e.

in an adapted coordinate system (U, u) with $u = (\sigma^\mu, x^i)$

$$\forall X, Y \in \Gamma_p(\pi), \quad X(p) = Y(p) \quad \text{and} \quad \left.\frac{\partial X^i}{\partial \sigma^\mu}\right|_p = \left.\frac{\partial Y^i}{\partial \sigma^\mu}\right|_p$$

The induced coordinate system (U', u') on $J^1\pi$ is defined by

$$U' = \left\{ j^1_p X : X(p) \in U \right\}$$
$$u' = \left(\sigma^\mu, x^i, x^i_\mu \right)$$

$x^i_\mu : U' \to \mathbb{R}$ are specified by $x^i_\mu = \left. \frac{\partial X^i}{\partial \sigma^\mu} \right|_p = \partial_\mu X^i \big|_p \equiv X^i_\mu \big|_p$

A general vector field in a complete basis of 1-jet space

$$V^{(1)} = V^\mu \frac{\partial}{\partial \sigma^\mu} + V^i \frac{\partial}{\partial x^i} + V^i_\mu \frac{\partial}{\partial x^i_\mu}$$

To preserve group action: For a vector field $V \in \mathfrak{X}(E)$,
$V = V^\mu \frac{\partial}{\partial \sigma^\mu} + V^i \frac{\partial}{\partial x^i}$, the prolongation of V is $V^{(1)} \in \mathfrak{X}(J^1\pi)$

$$V^{(1)} = V^\mu \frac{\partial}{\partial \sigma^\mu} + V^i \frac{\partial}{\partial x^i} + \left(\frac{dV^i}{d\sigma^\mu} - x^i_\nu \frac{dV^\nu}{d\sigma^\mu} \right) \frac{\partial}{\partial x^i_\mu}$$

where $\frac{d}{d\sigma^\mu} = \frac{\partial}{\partial \sigma^\mu} + x^i_\mu \frac{\partial}{\partial x^i}$

Consider now the following vector field prolongation

$$V^{(1)} = \Lambda^\mu \frac{\partial}{\partial \sigma^\mu} + \rho^i \frac{\partial}{\partial X^i} + \xi^i_\mu \frac{\partial}{\partial X^i_\mu}$$

The field transformations generated by $V^{(1)}$

$$\delta X^i = \rho^i_a \epsilon^a$$

$$\delta X^i_\mu = \xi^i_{\mu a}\epsilon^a = \left(\frac{d\rho^i_a}{d\sigma^\mu} - X^i_\nu \frac{d\Lambda^\nu_a}{d\sigma^\mu}\right)\epsilon^a = \left[\left(\partial_\mu \rho^i_a + X^j_\mu \frac{\partial \rho^i_a}{\partial X^j}\right) - X^i_\nu\left(\partial_\mu \Lambda^\nu_a + X^k_\mu \frac{\partial \Lambda^\nu_a}{\partial X^k}\right)\right]\epsilon^a$$

Set $\Lambda^\mu = 0$ and $\rho^i_a = \rho^i_a(X^i)$:

$$\delta X^i = \rho^i_a \epsilon^a \qquad\qquad \delta X^i_\mu = \frac{\partial \rho^i_a}{\partial X^j} X^j_\mu \epsilon^a$$

It is the global symmetry of the action if and only if

$$(\mathcal{L}_\rho G)_{ij} + \xi^k_\mu \frac{\partial G_{ij}}{\partial X^k_\mu} = 0 \quad \Leftrightarrow \quad \mathcal{L}_{V^{(1)}} G = 0$$

$$(\mathcal{L}_\rho B)_{ij} + \xi^k_\mu \frac{\partial B_{ij}}{\partial X^k_\mu} = \partial_{[i}\beta_{j]} \quad \Leftrightarrow \quad \mathcal{L}_{V^{(1)}} B = \mathrm{d}\beta$$

Global symmetries are Killing vector isometries of the 1-jet space of the mapping space

Comment

These symmetries can be enriched in another theories. For example, a Galilean symmetry $X^i_\mu \to X^i_\mu + b^i_\mu$ can be written as

$$\delta X^i = \rho^i(\sigma^\mu, X^j) = \hat{\rho}^i_a(X^j)\epsilon^a + \tilde{\rho}^{i\mu}_j(\sigma^\nu) b^j_\mu$$

Anomalies and T-duality

Now let's turn on background fields A^a, $\hat{A}_a$, with the bg gauge symmetry

$$\delta A^a = \epsilon^a \qquad \delta \hat{A}_a = \hat{\epsilon}_a$$

They couple through the currents

$$J_a^{\text{mom}} = (\iota_{\rho_a} G)_j F^j + (\iota_{\rho_a} B - \beta_a)_j * F^j \quad \text{and} \quad J_{\text{win}}^a = \delta_i^a * F^i$$

Under bg gauge transf.

$$\delta S = \int_{\Sigma_2} \delta_i^a \rho_b^i \epsilon^b \mathrm{d}\widehat{A}_a$$

Choosing adapted coordinates along the isometry directions s.t. $\rho_a^i = \delta_a^i$, we get the anomalous term from the inflow of 4D anomaly polynomial

$$\mathcal{I}_4 = \int_{\Sigma_4} \mathrm{d}A^a \wedge \mathrm{d}\widehat{A}_a$$

– There is a possibility of dyonic gauging

Goal 2: Generalize to p-form field and mixed tensor field theories

Target space structure

Consider multiple 1-forms A^i, with $i = 1, \ldots, D$

$A : T\Sigma_4 \to \mathcal{M} \qquad \dim \mathcal{M} = D$

– $\mathcal{M}$ is a graded manifold, coordinates a^i on $\mathcal{M}$ has deg $= 1$, and their pullback through the map A yield the spacetime 1-forms, $A^*(a^i) = A^i$

The action is a theory of such maps

$$S[A] = -\frac{1}{2}\int_{\Sigma_4} \left(G_{ij}(A, \mathrm{d}A)\, \mathrm{d}A^i \wedge * \mathrm{d}A^j + B_{ij}(A, \mathrm{d}A)\, \mathrm{d}A^i \wedge \mathrm{d}A^j \right)$$

Comments:

1) Effective actions for scalar Nambu-Goldstone bosons are nonlinear sigma models. Free photons can be seen as the Nambu-Goldstone boson for spontaneously broken 1-form symmetry (Cordova, Dumitrescu, Intriligator (2018))

2) Gauge invariant (BI, ModMax) vs non-gauge invariant theories

Comment on duality symmetry group

For the action

$$S[A] = -\frac{1}{2}\int_{\Sigma_4} \left(G_{ij}\,\mathrm{d}A^i \wedge *\,\mathrm{d}A^j + B_{ij}\,\mathrm{d}A^i \wedge \mathrm{d}A^j\right)$$

Upon dualization G_{ij}, B_{ij} transform to $\tilde{G}_{ij}, \tilde{B}_{ij}$ s.t. generalized complex metric $\tau_{ij} = B_{ij} + iG_{ij}$ transform as

$$\widetilde{\tau}_{mn} = \tau_{mn} - \tau_{ma}\tau^{ab}\tau_{bn}\,, \quad \widetilde{\tau}_{ma} = -\tau_{mb}\tau^{ba}\,, \quad \widetilde{\tau}_{ab} = -\tau^{ab}$$

Fractional linear transformation of τ_{ij} under duality group $Sp(2d, \mathbb{R})$.

This is the duality group for $(p = 2k - 1)$-form in (2p+2) dimensions

Back to the general action:

Let's consider a vector bundle (E, π, Σ) over the 4D spacetime Σ

(U, u) with $u = (\sigma^\mu, a^i)$ is a coordinate system on E

U is an open neighbourhood around a section of the tensor product bundle on E around a point $p \in \Sigma$.

1-jet tensor product bundle is defined s.t. the induced coordinate system (U', u') on $J^1\pi$ is given by

$$U' = \{j^1_p A^i : A^i(p) \in U\}$$
$$u' = (\sigma^\mu, a^i, b^i)$$

with b^i coordinates of degree two and, with pullbacks being implicit,

$$a^i : U \to \mathbb{R}, \qquad a^i = A^i\big|_p$$

$$b^i : U' \to \mathbb{R}, \qquad b^i = \frac{\partial A^i}{\partial \sigma^\mu}\bigg|_p d\sigma^\mu$$

Consider a Lie algebra valued graded vector field $V = V_a t^a$ on $\mathfrak{X}(E)$, $V_a = \rho^i_a \frac{\partial}{\partial a^i}$ with degree(V_a) = -1

Vector prolongation of V $\to V^{(1)} \in \mathfrak{X}(J^1\pi)$

$$V^{(1)}_a = \rho^i_a \frac{\partial}{\partial a^i} + \xi^i_a \frac{\partial}{\partial b^i} \qquad \text{with} \qquad \xi^i_a = b^j \frac{\partial \rho^i_a}{\partial a^j}, \qquad \deg(\xi^i_a) = 1$$

The graded vector field $V^{(1)}_a$ generates a 1-form global shift symmetry under

$$\delta A^i = \rho^i_a \epsilon^a \quad \text{and} \quad \delta F^i = \xi^i_a \epsilon^a \qquad (\xi^i_a = F^j \frac{\partial \rho^i_a}{\partial A^j})$$

ϵ^a: 1-form parameters

This is the global symmetry of the action iff

$$\mathcal{L}_{V^{(1)}_a} G = 0 \qquad \mathcal{L}_{V^{(1)}_a} B = 0$$

$\widehat{\mathcal{L}}_{V^{(1)}_a}$ is the jet space graded Lie derivative

We define a convenient degree:

Total degree = grading degree - vector degree

$$|\frac{\partial}{\partial a^k}| = -2\,, \quad |\frac{\partial}{\partial b^k}| = -3\,, \quad |\rho^i_a| = 0\,, \quad |\xi^i_a| = 1$$

For $|V_a^{(1)}| = p$ and $V_1, V_2 \in \mathfrak{X}(J^1\pi)$ graded vect. of tot. deg. $=k$

$$(\widehat{\mathcal{L}}_{V_a(1)} G)(V_1, V_2) = V_a(1)(G(V_1, V_2)) - G(\widehat{\mathcal{L}}_{V_a(1)} V_1, V_2) - \alpha^{k+2}\, G(V_1, \widehat{\mathcal{L}}_{V_a(1)} V_2)$$

In the case in hand $|V_a^{(1)}| = |V_1| = |V_2| = 2$

– B, G symmetric 2-tensors $\rightarrow$ generalized target space metrics

Generalized global shift symmetries of any 4D nonlinear gauge-invariant theory of 1-forms $\sim$ Killing isometries for generalized (1-jet extension) target space background

Anomalies in 1-form theories

As an example, let's consider

$$S[A] = \int \mathcal{L}[\mathrm{k}(F), \mathrm{t}(F)]\, \sqrt{|g|}\, \mathrm{d}^4\sigma$$

with $\mathrm{k}(F) = \frac{1}{2} * (F \wedge *F)$ and $\mathrm{t}(F) = \frac{1}{2} * (F \wedge F)$

Maxwell theory: $\mathcal{L} = k$,
BI: $\mathcal{L} = -1 + \sqrt{1 + 2\,\mathrm{k} - \mathrm{t}^2}$
ModMax: $\mathcal{L} = \cosh(\gamma)\, \mathrm{k} + \sinh(\gamma)\, \sqrt{\mathrm{k}^2 + \mathrm{t}^2}$

The currents are

$$J_{\text{ele}} = \partial_{\mathrm{k}}\mathcal{L}\, F - \partial_{\mathrm{t}}\mathcal{L} * F \qquad\qquad J_{\text{mag}} = *F$$

Couple to bg fields

$$S[A, B, \widehat{B}] = \int \mathcal{L}[\mathrm{k}(F, B), \mathrm{t}(F, B)]\, \sqrt{|g|}\, \mathrm{d}^4\sigma - \int_{\Sigma_4} \widehat{B} \wedge F$$

$$\delta S = -\int_{\Sigma_4} \epsilon \wedge \mathrm{d}\widehat{B} \longrightarrow \mathcal{I}_6 = -\int_{\Sigma_6} \mathrm{d}B \wedge \mathrm{d}\widehat{B}$$

Outline

Fierz-Pauli action for a massless graviton $h_{\mu\nu}$ in a flat 4D background

$$S[h] = -\frac{M_P^2}{2g^2}\int \mathrm{d}^4x\,(\partial^\mu h^{\nu\rho}\partial_\mu h_{\nu\rho} - 2\partial^\mu h^{\nu\rho}\partial_\nu h_{\mu\rho} + 2\partial^\mu h^\nu{}_\nu\partial^\rho h_{\rho\mu} - \partial^\mu h^\nu{}_\nu\partial_\mu h^\rho{}_\rho)$$

Curvature tensor $R_{\mu\nu\rho\sigma}$ is dualized on $\begin{cases} \text{the first slot} & *R \\ \text{the second slot} & R* \\ \text{both slots} & *R* \end{cases}$

Define a 2×2 matrix of tensors $\bar{R}_{ij} = \begin{pmatrix} R & *R \\ R* & *R* \end{pmatrix}$ $\quad i,j = 1,2$

Duality relation $\bar{R}_{ij} = J_i{}^k(*\bar{R})_{kj}$ $\quad J = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$

Expanding Duality relation

$$\begin{cases} \widehat{R} = \frac{1}{g^2} * R - \theta R \\ \check{R} = 2\tau_1\tau_2 * R - (\tau_1^2 - \tau_2^2)R \end{cases} \qquad \tau = \tau_1 + i\tau_2 = \theta + \frac{i}{g^2}$$

Linearized Gravity

Graded Geometry

Graded (super)manifold: (Chatzistavrakidis, Karagiannis, Peter Schupp (2020))

1) Two sets of anticommuting (degree-1) coordinates θ^μ and $\widetilde{\theta}^\mu$

$$\{\theta^\mu, \theta^\nu\} = 0\,, \quad \{\widetilde{\theta}^\mu, \widetilde{\theta}^\nu\} = 0\,, \quad [\theta^\mu, \widetilde{\theta}^\nu] = 0$$

$\theta^\mu \equiv (1,0) - \mathrm{degree}$ $\qquad$ $\widetilde{\theta}^\mu \equiv (0,1) - \mathrm{degree}$

Any function is an expansion on the space of graded coordinates

E.g. Graviton: $h = h_{(\mu\nu)}(x)\theta^\mu \widetilde{\theta}^\nu$

2) Two homological vector fields: $\mathrm{d} = \theta^\mu \frac{\partial}{\partial x^\mu}$ and $\widetilde{\mathrm{d}} = \widetilde{\theta}^\mu \frac{\partial}{\partial x^\mu}$

3) Generalized Hodge star $\star\omega$ [degree-$(d-p, d-q)$ function]:

$$\star\omega = \frac{1}{(d-p-q)!}\, \eta^{d-p-q}\, \widetilde{\omega}$$

$$\star\omega = *\,\widetilde{*}\,(-1)^{\epsilon(p,q)} \sum_{n=0}^{\min(p,q)} \frac{(-1)^n}{(n!)^2}\, \eta^n\, \mathrm{tr}^n\, \omega$$

4) Trace operator: $\mathrm{tr} = \eta^{\mu\nu} \frac{\partial}{\partial\theta^\mu} \frac{\partial}{\partial\widetilde{\theta}^\nu}$

5) Co-trace operators: $\sigma = -\theta^\mu \frac{\partial}{\partial\widetilde{\theta}^\mu}$ and $\widetilde{\sigma} = -\widetilde{\theta}^\mu \frac{\partial}{\partial\theta^\mu}$

Fierz-Pauli action:

$$S[h] = -\frac{M_{\mathrm{P}}^2}{4g^2} \int \mathrm{d}^4x \, \mathrm{d}^4\theta \, \mathrm{d}^4\widetilde{\theta} \, \mathrm{d}h \star \mathrm{d}h \equiv -\frac{M_{\mathrm{P}}^2}{4g^2} \int_{\hat{\Sigma}_4} \mathrm{d}h \star \mathrm{d}h$$

Linearized Riemmann tensor: $R = \mathrm{d}\widetilde{\mathrm{d}}\, h$

Linearized Einstein eqs: $\mathrm{tr}\, R = 0$

Linearized Bianchi identities: $\mathrm{d}R = 0$ and $\widetilde{\mathrm{d}}R = 0$

Gravitational duality in 4D $\sim$ Twisted self-duality

$$\widehat{R} = \frac{1}{g^2} * R$$

Irreducibility of $\omega_{p,q}$ under GL group $\sim$ $\sigma\omega = 0$ for $p \geq q$

Under duality $\begin{cases} \sigma R = 0 & \leftrightarrow \quad tr\widehat{R} = 0 \\ trR = 0 & \leftrightarrow \quad \sigma\widehat{R} = 0 \end{cases}$

Duality invariant action (Parent action):

$$\mathfrak{L}[f,\lambda] = -\frac{M_P^2}{4}\int_{\theta,\widetilde{\theta}} f\star\left(\frac{1}{g^2} + \theta *\right)\mathcal{O}f - 2\int_{\theta,\widetilde{\theta}} f * \widetilde{*}\, \mathrm{d}^\dagger\lambda \qquad \mathcal{O} = \mathrm{id} - \frac{1}{2}\widetilde{\sigma}\sigma$$

EOM for λ : $\mathrm{d}f = 0 \quad \to \quad f = de, \qquad e = h + \sigma b$

$\mathfrak{L}[h] = -\frac{M_P^2}{4}\int_{\theta,\widetilde{\theta}} \left(\frac{1}{g^2}\mathrm{d}h\star\mathrm{d}h - \theta\,\mathrm{d}h\tilde{*}\mathrm{d}h\right) + \text{bdy terms}$

EOM for f : $\quad \star\left(\frac{1}{g^2} + \theta *\right)\mathcal{O}f = -\frac{4}{M_P^2} * \widetilde{*}\,\mathrm{d}^\dagger\lambda$

Hull's duality relation: $\widehat{R} = \frac{1}{g^2} * R - \theta R$

$\mathfrak{L}[f,\lambda] \to \tilde{\mathfrak{L}}[\widehat{h}] \qquad \tau \to -\frac{1}{\tau}$

Linearized Gravity

Comment (1): In the presence of theta-term, original and double dual gravitons are independent.

Comment (2): Theta-term is linearized Nieh-Yan topological invariant

$$N = T^a \wedge T_a - R_{ab} e^a \wedge e^b = d(e^a \wedge T_a)$$

In 4D, NY term is the only closed 4-form invariant under local Lorentz rotations associated with the torsion of the manifold

$\int_{M_D} N \propto P[SO(D+1)] - P[SO(D)]$

In local coordinates: Linearized NY $\propto \epsilon^{\mu\nu\rho\sigma} \partial_\mu h_{\nu\kappa} \partial_\rho h^\kappa_\sigma$

Linearized Gravity

Off-shell action for p-form and (p,1)-form theories

$$\mathfrak{L}[F^M_{p+1,q}, \Lambda_{p+2,q}] = (-1)^{p+q}\alpha^2 \left[\int_{\theta,\widetilde{\theta}} F^M \star \mathcal{U}_{MN}\mathcal{O}F^N - 2\int_{\theta,\widetilde{\theta}} F * \widetilde{*}\, \mathrm{d}^\dagger \Lambda\right], \quad q = 0, 1$$

$$\mathcal{U}_{MN} = G_{MN} + (-1)^{p+q} B_{MN} *$$

$$\mathcal{O} = \begin{cases} \mathrm{id}\,, & q = 0\,, p \geq 0 \\ \mathrm{id} - \frac{1}{p+1}\widetilde{\sigma}\sigma\,, & q = 1\,, p \geq 1 \end{cases}$$

$$S[A^M] \xleftarrow{\Lambda \text{ on-shell}} \mathcal{S}[F^M, \Lambda] \xrightarrow{F \text{ on-shell}} \widetilde{S}[\widehat{A}^M]$$

How about generalized global symmetry and anomaly in linearized gravity?

Can we see the off-shell action of linearized gravity as the action for Nambu-Goldstone bosons (gravitons) for a broken (1,1)-symmetry?

Is there an analouge of Coleman-Mermin-Wagner theorem for p-form (Lake 2018, Hofman1, Iqbal 2017,2019) and (p, q)-mixed tensor field theories?

Summary

- T-duality group in $d = 2p + 2$ dimensions
$$\begin{cases} O(d,d;\mathbb{R})\,, & p = 2k \text{ scalar, 2-form, Curtright} \\ Sp(2d;\mathbb{R})\,, & p = 2k-1 \text{ 1-form, graviton, (p,1)-form} \end{cases}$$
- Off-shell duality invariant parent action for p-forms and (p,1) mixed tensor theories
- Off-shell duality of “non-linear” theories
- Generalized global symmetries as Killing isometries of generalized (jet space extension of) graded target space
- t’ Hooft anomalies of generalized global symmetries and anomaly matching in T-dual picture
- Nieh-Yan invariant as the origin of topological gravitational theta-term

In the future

- Whenever there are both 0-form and higher-form symmetries present, there is a notion of n-group symmetry. (Benini, Cordova, Hsin (2019))
  It can be understood from the junction of symmetry defects.

Q: What's the symmetry defect in the gravity (or in general a mixed tensor) case and how it's related to the n-group?

- How does everything pan out for discrete symmetries? Non-invertible symmetries?

Thanks!